MODELLING INNOVATIVE REGIONS:
CONCEPTUAL, MATHEMATICAL AND COMPUTATIONAL CHALLENGES

Invited Background Paper

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Dr Brett Parris

Research Fellow
Department of Econometrics & Business Statistics
Monash University
&
Chief Economist
World Vision Australia

Tel: + 61 403 866 244
Brett.Parris@buseco.monash.edu.au
Brett.Parris@worldvision.com.au

1. Introduction

This Dahlem Workshop on the mathematics of social entities could hardly have come at a more opportune time. Two previous Dahlem workshops (Schellnhuber *et al.* 2004 and Costanza *et al.* 2007) developed the themes of earth systems modelling and sustainability. These earlier workshops provided a solid foundation for this workshop seeking a greater understanding of the mathematical models of social and economic systems that are needed to complement the comparatively well-developed earth system and climate models.

Following the *Fourth Assessment Report* of the Intergovernmental Panel on Climate Change in 2007 and the *Millennium Ecosystem Assessment* in 2005, there is a growing realisation that the ecosystems supporting our economies and societies are under enormous stress, placing the future of our civilization at risk. In September 2008, the prominent Australian economist Ross Garnaut released his review of climate change for the Australian Government, concluding with a justifiably dramatic flourish (Garnaut, 2008, p. xlv): “On a balance of probabilities, the failure of our generation on climate change mitigation would lead to consequences that would haunt humanity until the end of time.”

The International Energy Agency (IEA) meanwhile warned that while global energy demand continues to grow strongly, “A supply-side crunch in the period to 2015, involving an abrupt escalation in oil prices cannot be ruled out” (IEA, 2007, p. 43). In an interview in *Time* magazine on 7th November 2007, the IEA’s Chief Economist, Fatih Birol, bluntly warned that the world’s oil supplies were much more vulnerable than most people realised, due to inadequate investment: “I am sorry to say this, but we are headed toward really bad days … Lots of targets have been set but very little has been done”. The past year has also witnessed a food price crisis thrusting an estimated 100 million people back into poverty and leading to riots in more than 30 countries, and a global financial crisis that brought the international financial system to the brink of the abyss.

Innovation is clearly essential to help us overcome the social, economic and environmental challenges we are facing – and innovation not just in technologies, but in policies, in institutions, in legislation, in international cooperation and in social norms. But what kinds of mathematical and computational models are best suited model these innovations, to help guide decision making on these challenges? This background paper on innovative regions explores some of these themes, with a particular emphasis on the modelling of technological innovation in environment-economy models. Since this is a background paper intended to spur discussion, I have chosen to canvas a large number of issues without going into too much detail on any. The outline of the paper is as follows: Section 2 raises the question of why we are choosing to focus on the ‘mathematics of social entities.’ Section 3 outlines some of the most common pitfalls in mathematical and computational economic modelling. Section 4 focuses more specifically on technological innovation, briefly canvassing a large number of issues to be considered in modelling innovative regions. Section 5 introduces agent-based modelling and its potential for modelling innovative regions. Section 6 discusses the potential for the further development of hybrid modelling. Section 7 raises questions about the possible trade-off between quantitative and qualitative accuracy. Section 8 concludes with some reflections on the place of modelling in policy development.

2. The 'mathematics of social entities' - for what purpose?

Why focus on the mathematics of social entities? What is our aim? It is merely to indulge an academic interest, far removed from the often harsh realities of the social systems we study? Or do we seek understanding in order to help improve outcomes for the people living in those systems? The question may seem impertinent, but over the course of this paper I will argue that the extent to which our models are anchored to the real world has a great influence over how we understand the role of mathematical modelling in policy development.

I would like to suggest that our primary aim in studying the mathematics of social entities should be to use the power of mathematics and computation to better understand real-world social and economic systems. In any science there is an important role for basic theoretical research, but given the enormous challenges facing our societies, those of us privileged to have been given such opportunities should ensure that both our theoretical and applied research maintains close contact with the real-world socio-economic systems we are supposedly studying. At times, economics has drifted far from its real-world moorings, with extraordinary efforts devoted to the study of axiomatic mathematical 'economies' which yield little insight into actual economic systems.

Robert Clower (1995, p. 317) expressed this concern well: "I do not in any way mean to denigrate the intellectual excellence of neowalrasian proofs of competitive equilibrium; I intend only to suggest that this work is more accurately categorized as set-theoretic logic than economics." In other words, while the variables in the Arrow-Debreu general equilibrium model that formed the foundation for so much economic theory may have economic names, the model is actually more like axiomatic mathematical philosophy, virtually useless for improving our understanding economic systems in the real world. This observation led Clower (1995, p. 317) to conclude: "I find no logical flaw in any aspect of Arrow-Debreu theory; I argue, however, that as a foundation for applied economics, Arrow-Debreu theory is empirically vacuous and conceptually incoherent."

In seeking to better understand the mathematics of social entities, I would venture to suggest that the opportunity cost of such rarefied mathematical exercises is too high. We need more of our best minds focussed on mathematical and computational models that actually help to improve the circumstances of those at the bottom of our social systems and which help our economies to shift to an environmentally sustainable path. That will only come from a deeper understanding of real-world socio-economic systems and their complex, dynamic interactions with the earth-biosphere-climate system.

3. Common pitfalls in mathematical and computational economic models

Given the dominance of economics in so much public policy discourse, and the seriousness with which economists' pronouncements are taken, it can come as a surprise to those schooled in other disciplines to learn that so many of the economic theories and models still routinely employed by economists have received devastating criticisms from fellow economists in the mainstream academic economic literature.

In his book *Economic Models of Climate Change: A Critique,* Stephen DeCanio (2003) provides an incisive analysis, drawn principally from the mainstream economic literature, of the pitfalls of certain aspects of economic theory and of models built upon a simplistic application of that theory. DeCanio warns for example (pp. 6-7):

> As we shall see, the simplifications of neoclassical economics strip away essential information about the system, not just the inessential accidentals. The consequences for climate policy have been severe. … [T]he application of general equilibrium analysis to climate policy has produced a kind of specious precision, a situation in which the assumptions of the analysts masquerade as results that are solidly grounded in theory and data. This leads to a tremendous amount of confusion and mischief, not least of which is the notion that although the physical science of the climate is plagued by uncertainties, it is possible to know with a high degree of certainty just what the economic consequences of alternative policy actions will be. This myth, more than any other, has created the policy paralysis and public confusion that have so far impeded constructive action … to meet the climate challenge.

Analytic mathematical models such as Optimal Growth (OG) and Computable General Equilibrium (CGE) models are the standard work-horses for much economic policy analysis. There are however some very well-known and serious problems with these models that have been described in the economic literature over the past several decades. Here is not the place for a full elaboration of these issues, but several limitations of these approaches should be mentioned before discussing technological innovation more specifically in the next section:

- They usually rely on a '**representative agent**' to represent the population, but the preferences and welfare changes accruing to a representative agent may have little in common with the preferences and welfare changes of a population of heterogeneous individuals (Kirman, 1989, 1992; Rizvi, 1994).

- The OG and to a lesser extent CGE models are **highly aggregated**, often with insufficient attention to the mathematical complexities of aggregation across heterogeneous categories and aggregation between different **scales** in complex social and economic systems (Blundell & Stoker, 2005). Jaeger and Tol (2002, p. 151) rightly emphasised the importance of understanding how different scales interact: "steps towards sustainable development must also be taken at the spatial scales of regions and at the temporal scales of individual lives. …. Understanding how processes at the regional, national and global level interact in the short and in the long run will be vital for a successful management of the transition towards a sustainable world economy."[1]
- They must rely on very strong assumptions to exclude **multiple equilibria and chaotic dynamics**, when in fact these are common in all but the most restrictive analytic models (Kehoe, 1985, 1998; Saari, 1995, 1996).
- They usually presume **perfect information**, when in fact information imperfections and uncertainty are pervasive in real economies, affecting everything from the type and function of institutions to the development of credit and insurance markets (Stiglitz, 2002). Stiglitz (p. 480) was scathing of the representative agent approach for example, because it "by construction, ruled out the information asymmetries which are at the heart of macroeconomic problems".
- They are generally 'real' models, effectively **barter models**, without financial, credit, risk and insurance markets – all of which are crucial to modelling economic dynamics, including technological innovation (Schumpeter, 1934; Dillard, 1988).
- They generally employ notions of '**capital'** which remain controversial more than 50 years after the 'capital controversies' erupted within the economics profession due to mathematical problems with aggregating across heterogeneous 'capital' categories and the fact that the value of that capital is inseparable from the interest rate, meaning that defining the interest rate as the marginal product of capital is problematic (Cohen & Harcourt, 2003).
- CGE models are generally **comparative static** models, rather than truly dynamic evolutionary models. Even so-called 'dynamic' CGE models are usually just a series of comparative-static steps. But comparative statics is just a mathematical exercise that by using highly restrictive mathematical assumptions, engineers a unique equilibrium. It has nothing to say about the disequilibrium transition dynamics that an economy must undergo once a real-world equilibrium (if such a beast exists) is disturbed, and there are no sound theoretical reasons to believe that the new equilibrium posited by comparative static analysis could actually be found once the system

[1] See also the whole special issue on scaling issues in *Integrated Assessment*, 2002, Vol. 3, No. 2 & 3.

entered a state of disequilibrium (Kehoe, 1985; Fisher, 2003). After years of reflection Stiglitz (2002, pp. 486-487), mused in his Nobel lecture: "I have become convinced that the dynamics of change may not be well described by equilibrium models that have long been at the center of economic analysis. … Dynamics may be better described by evolutionary processes and models than by equilibrium processes."

- They generally use smoothly differentiable **production functions** which imply perfect substitutability between inputs and all possible combinations. Real world technologies however, are discrete or 'lumpy' with only certain combinations of inputs technically possible. This non-substitutability of inputs is critical for modeling technological development (Felipe & Fisher, 2003; Ayres, 2008).
- The **expectations-formation processes** within the models are usually primitive, relying on a long-discredited 'rational expectations' approach that presumes perfect foresight. But the formation of expectations about the future is a crucial dynamic affecting revaluation of existing assets as well as consumption, saving and investment decisions (Arrow, 1987).
- The **modelling of firms** is frequently undertaken in terms of a single 'representative firm' representing an entire industry. This makes no allowance for firm heterogeneity and it presumes that all firms are on the technological frontier – so by definition there are no gains to be had from further efficiencies – no 'no regrets' initiatives to be had. Firms are also usually assumed to follow a basic profit-maximization model that bears little relation to the more sophisticated modern behavioural theories of the firm. As a result, OG and CGE models are also generally poor at modelling processes of genuine innovation and creativity – particularly those involving adaptive responses to government policies, incentives and disincentives (Nelson & Nelson, 2002; Dew *et al.* 2008).
- They generally presume **complete markets** for all goods, services, land, labour, capital and risk (if the latter is considered at all). These complete markets are founded upon the *existing* structure and distribution of rights, particularly property rights – a step with its own ethical implications. But incomplete and spatially and temporally separated markets are a pervasive feature of real economies. Relaxing the assumption of complete markets generally undoes the notion of a unique equilibrium so that multiple equilibria prevail (Arrow, 1987, pp. 72-73).
- They use systems of equations that presume that all stocks and flows of money or value are conserved. But value does not necessarily follow **conservation principles** in real-world economic systems. Changed expectations can wipe billions from the value of stock markets overnight with prices jumping discontinuously without any change in underlying fundamentals and without the value 'going somewhere' except back where it came from – people's minds. Prices (and hence value) can also jump discontinuously in value chains as a result of asymmetric

bargaining power between buyers and sellers. The failure of value to obey conservation principles has led some (e.g. Northrop, 1941) to deny the possibility of a theoretical mathematical economic dynamics.

- There is **more than one kind of mathematics**. Almost all economic models simply presume that real analysis (mathematical analysis over the real number domain) is appropriate. But Vela Velupillai (2005) has challenged this presumption repeatedly, arguing instead that since prices and quantities are denoted in integers, economic agents must undertake integer optimization. Moreover, it is not enough simply to demonstrate the existence of an equilibrium mathematically. For it to have any real-world significance, it must be computable – able to be constructed. Velupillai concludes that economic models should use computable or constructive mathematics, not real analysis. If agents try to optimize, it must be combinatorial integer optimization. An important feature of such integer (or Diophantine) problems is that it is not necessarily possible to know in advance whether a Diophantine equation has a solution.[2] Therefore it is not possible to optimize the amount of resources devoted to searching for a solution. This mitigates against any *a priori* assumption that a global optimum is achievable.
- They almost invariably presume **perfectly rational agents** with infinite computational capacities. It is now well known however that agents with 'bounded rationality' are essential to better approximate real world behaviours (Conlisk, 1996).

In short, the types of mathematical scenarios that agents face are critical: There is an enormous difference between optimization by perfectly rational agents with perfect information under real analysis in a stable system, and integer optimization by boundedly rational agents operating under imperfect information in a complex, evolving, spatial system. There is no theoretical reason whatsoever to believe that the first will even be a remote approximation of the second.

The economics profession is starting to move beyond the restrictive analytic mathematical models of the past represented by OG, CGE and more recently Dynamic Stochastic General Equilibrium (DSGE) models (Colander, 2006). By using simulation models, it is starting to recover some of the earlier discursive insights that were too difficult to model analytically as well as exploring new territory (Colander *et al.* 2008).

[2] This statement follows from the solution to mathematician David Hilbert's tenth problem (from a list of 23 problems presented at the 1900 International Mathematical Congress in Paris) that was solved by Yuri Matiyasevich in 1970.

4. Modelling Innovation and Technical Change

4.1 Country experience with innovation and technical change

Most of today's industrialised countries followed some form of industry promotion and technological deepening strategy. Economists debate the wisdom, costs and benefits of these initiatives, but it is clear that there are few historical examples of national technological deepening simply being left to the vagaries of the free market. The United States and Germany for example, did not listen to the 19th century British economists telling them they should specialise in their then-current comparative advantages in agriculture, so as to leave the serious manufacturing to Britain. More recently, the governments of the newly industrialised countries such as Korea and Taiwan also used a battery of tariffs, quotas, standards, subsidies and other incentives to strengthen the technological bases of their economies (Kim & Nelson, 2000).

The burgeoning literature on technological change both in the industrialised and developing regions has shown that an extensive range of measures have been deployed by governments, and that the details of firm-government interactions and how these affect the incentives of firms and investors are critical to understanding the processes of technological change (Rodrik, 2005). Simplistic aggregate-level frameworks such as considering countries to be either 'open' versus 'closed', or 'free market' versus 'state controlled' do not help us understand the detailed policies that have enabled some countries and regions to become technological leaders while others have languished.

4.2 Modelling innovation and technical change

In recent years there has been an outpouring of publications related to modelling innovation and technical change, particularly in the context of economy-energy-environment interactions. Important recent works include:

- The special issue of *Energy Economics* on the Stanford Energy Modeling Forum[3] EMF 19 Study on Technology and Global Change Policies (Weyant, 2004);
- The *Energy Journal* special issue on *Endogenous Technological Change and the Economics of Atmospheric Stabilization* , reporting the results of the Innovation Modeling Comparison Project (IMCP)[4] (Grubb *et al.* 2006; Köhler *et al*. 2006; Edenhofer *et al*. 2006);
- The *Energy Journal* special issue on *Hybrid Modeling of Energy-Environment Policies* (Hourcade *et al*. 2006);

[3] See: http://www.stanford.edu/group/EMF/
[4] See: http://www.econ.cam.ac.uk/research/imcp/

- The special issue of *Energy Economics* on Technological Change and the Environment (Fisher-Vanden, 2008; Gillingham *et al.* 2008; Pizer & Popp, 2008);
- Individual surveys such as Grubb *et al.* (2002) and Löschel (2002) and books such as Grübler *et al.* (2002).

An (incomplete) database constructed for this background paper synthesizing much of this literature lists more than 70 models currently being used in this field. The models can be classed broadly into six categories:

Optimal growth (OG) models aim to model long-term growth dynamics with intertemporal optimization of investor behavior. They tend to be highly aggregated top-down (TD) models and technological change may be either exogenous, determined by a parameter such as an Autonomous Energy Efficiency Index (AEEI), or endogenous, with relative price effects inducing technical change. Examples from the literature include DEMETER-1CCS, FEEM-RICE and MIND.

Computable General Equilibrium (CGE) models usually compute a comparative static demand-supply equilibrium. Purely comparative static models simply calculate a new equilibrium after a 'shock' to one of the key variables, with the new equilibrium presumed to represent the economy at a later period – how late depending on whether the 'closure' has been set up to represent the 'short-run' or 'long run'. Dynamic CGE models usually undertake a series of comparative static calculations one period at a time, and are referred to as 'recursive dynamic', in contrast to dynamic models in which results for all periods are computed (and optimized) simultaneously. CGE models are generally more disaggregated than OG models, with numerous commodities, industries and geographic regions, and are best at indicating likely relative price effects in response to economic shocks. Most rely on representative households, though some have been linked with microsimulation models that include multiple households. Examples from the literature include MMRF, IMACLIM-R, WORLDSCAN and AMIGA.

Macroeconometric (ME) models are systems of differential equations with parameters estimated from time series data. ME models begin with a certain time slice of the economy and calculate subsequent steps. They are typically rich in bottom-up detail for particular sectors. Examples from the literature include EGEM, E3ME and E3MG.

Energy system (ES) models "usually derive a cost-minimum sequence of energy technologies for an exogenously given energy demand using linear programming. In more advanced versions, the energy technologies are improved by learning- by-doing. The main advantages of this approach are the detailed depiction of the energy sector and the possibility of basing technological change on an engineering assessment of different technologies" (Edenhofer et al. 2006, p. 66). The main disadvantage of ES models is that they are partial equilibrium models which do not take into account the economic repercussions on other sectors of the economy. Examples from the literature include MARKAL, NEMS, POLES, DNE21+ and GET-LFL.

Integrated Assessment (IA) models seek to integrate both biophysical and socio-economic systems. Due to computational constraints one or both aspects of the model tend to be fairly simple representations of the different systems. Examples from the literature include ETC-RICE and ICAM3.

Hybrid models link different classes of models together, frequently one that is more top-down, such as an OG or ME model, and one that is more bottom up, such as an ES model. Examples include MESSAGE-MACRO, and MARKAL-MACRO, in both cases linking an ES model with the MACRO macroeconomic model.

Technological Change (TC) itself may be modeled as:

- **Exogenous Technical Change –** imposed on the model by means of a pre-defined parameter, such as the Autonomous Energy Efficiency Index (AEEI). The use of an AEEI is particularly common in top-down OG models.
- **Endogenous Technical Change (ETC) –** where economic forces within the model itself shape technological change. Most modern models incorporate some form of ETC.
- **Induced Technical Change (ITC) –** where deliberate policies induce technical change above or below a business-as-usual ETC baseline scenario.[5]

4.3 Representing technologies

One of the most important differences between models lies in whether the model represents technologies from a 'top-down' approach, using smoothly differentiable production functions (which were discussed briefly in Section 3) or an 'engineering' approach which specifies only certain discrete, technically- feasible combinations of equipment and inputs. The engineering approach is considerably more realistic, but also more data-intensive and computationally difficult for models that rely on systems of differential equations (Weyant, 2004, p. 507).

4.4 Research & Development (R&D)

Including R&D is one of the main means by which TC is endogenised and this is frequently undertaken in a fairly top-down manner by means of investments to add to a stock of knowledge which improves productivity. Some models separate public R&D from private R&D. The potential opportunity costs of increased R&D in one particular sector, say low-carbon energy technology, at

[5] Here I follow the definitions of Edenhofer *et al.* (2006), p. 58.

the expense of other sectors, is recognised as an important consideration that affects the cost-benefit calculations of particular policies. Increased publicly-funded R&D can lead to crowding-out effects which reduce R&D elsewhere. These crowding-out effects tend to increase the projected costs of climate change mitigation initiatives in models such as ENTICE-BR and FEEM-RICE. The seriousness of such crowding-out effects could be better assessed by explicitly modelling children's development and the education and training system so that R&D capacity is deliberately increased over time.

4.5 Learning-by-doing and learning curves

Modelling learning, particularly learning-by-doing where productivity improves with accumulated production, and by the incorporation of learning curves into the models is the other principal means by which TC is endogenised. Steep learning-by-doing curves are essential to rapidly bring down costs in new technologies (Fri, 2003, pp. 70-71).

4.6 Technological diffusion

For new innovations to have any significant impact they must diffuse through an economy. How such diffusion actually takes place has been the subject of considerable study (Keller, 2004), and a number of models now take seriously the need to include diffusion. The MESSAGE-MACRO and POLES models for example, include not only knowledge investment and learning curves, but also technology diffusion curves for different energy technologies (Grubb *et al.* 2002, p. 297).

4.7 Technological spillovers and industrial clustering

It is well known that technologies have spillover effects (both positive and negative), but technological spillovers are frequently neglected in models. Some models which do have some modelling of spillovers include ETC-RICE and MESSAGE-MACRO. Very few models however, include the spatial dynamics of industrial clustering, since most economic-energy-environment models are 'spatial' only to the extent of considering different countries and most do not model either spatial geography or heterogeneous individual firm behaviours. The IMCP project team explicitly mentioned regional spillovers as being a subject that warranted further exploration (Edenhofer *et al.* 2006, p. 105).

4.8 Increasing returns and path dependency

Increasing returns refer to both production systems in which the synergistic effects of increases in inputs lead to even greater increases in outputs, and also to the economy-wide effects of higher productivity in one sector leading to higher incomes, higher demand and higher investment in other sectors. ETC due to knowledge capital and experience curves leads to spillovers, learning and increasing returns, which in turn leads to imperfect competition and path dependency, including first-mover advantages (Köhler *et al.*, 2006, pp. 37-38; Arthur, 1994). Increasing returns present considerable mathematical challenges for analytic mathematical models, such as nonlinearities and non-convexivities, often making such models intractable. For these reasons, standard models such as IAMs assume constant returns to scale (Jaeger & Tol, 2002). As the IMCP team (Köhler *et al.*, 2006, p. 49-50) concluded:

> Recursive CGE models based on linear programming solutions face particular difficulties, because they may become unstable when they incorporate increasing returns. … Dynamic simulation models that already incorporate increasing returns and do not optimize are able to incorporate learning curves and increases in productivity from R&D. The bottom-up models, which are based on cost minimization, face similar problems to CGEs in including increasing returns, but have found various ways of overcoming the difficulties.

4.9 The role of expectations and decision making

How agents' expectations are modelled exerts a critical influence over model results since this determines whether investors act with very limited foresight, perfect foresight or anything in between. The IMCP team concluded that, "assumptions about long-term investment behavior have a strong impact on mitigation costs and strategies. Therefore, experiments with different assumptions about long-term expectations and long-term flexibility of investment behavior would be highly valuable" (Edenhofer *et al.* 2006, p. 105).

4.10 The mathematics of discount rates

The discount rates used for economic models also have a major influence on the policy recommendations flowing from model results, particularly in regards to recommendations on technological innovation policy. High discount rates, using market interest rates for example, effectively devalue the future, implying that strong emissions reductions and investments in innovation should be delayed as long as possible. Lower discount rates are used by those, such as Nicholas Stern (2008, p. 13), who argue that the future should not be devalued simply because it is in the future, and that there are no mathematical shortcuts to discovering some 'correct' discount rate – ethical considerations are also essential: "We come back again to a basic conclusion: the notions of ethics, with the choice of paths, together determine endogenously the discount rates. There is no market-determined rate that we can read off to sidestep an ethical discussion."

4.11 Modelling creativity and innovation

The processes of creativity and innovation have been extensively studied, and are now being distilled into their essential ingredients (Pahl *et al.*, 2007). While still in its infancy compared with other aspects of this field, the prospect of identifying the core ‘memes’, ideas, or basic actions common to virtually all innovation, potentially offers a tremendous opportunity for algorithmic combinations of these basic components, perhaps ‘growing’ innovations through evolutionary algorithms. Modelling innovation and creativity is likely to become considerably richer in the future than simply using learning curves.

4.12 Financing innovation – ‘capital’ and financial markets

Joseph Schumpeter (1934) emphasised the critical importance of credit markets to finance innovation and new technologies – yet most models have no financial sector or representation of credit markets and rely on problematic representations of ‘capital’ (discussed briefly in Section 3). The capital deployment decisions of firms however, are critical (Fri, 2003, p. 62), as are the interactions between natural disasters, finance and insurance markets. For example, in a detailed study of the Chesapeake Bay area in the United States, Michael (2007) showed that periodic inundations by storm surges are likely to be of the order of 9-28 times more expensive than permanent inundation, because of factors such as repeated rebuilding and repair costs and higher insurance premiums. Similarly, new scientific information on likely sea-level rises can have immediate effects on coastal property values, and hence loans and taxes linked to those property values, with repercussions throughout the financial system. Most models ignore these financial and insurance market effects and the ability (or not) of innovative firms to attract the finance necessary to commercialise new technologies.

4.13 The importance of heterogeneity and niche markets

Heterogeneous markets, and particularly niche markets, are important, since new technologies often find ‘early adopters’ in niche markets. As Fri (2003, p. 59) emphasised, ‘disruptive’ technologies need to be nurtured or matured in niche markets in order to improve enough to be ready for the ‘prime time’ of the mass market. New technologies often do not appeal to the existing demands of the mass market. Models with aggregate markets and representative firms can not represent niche markets where new technology may be seeded (Köhler *et al.* 2006, p. 48):

> The models compared in the IMCP are all deterministic. This is a critical limitation, because non-linear, dynamic systems with heterogeneous agents where responses are essentially stochastic have fundamentally different properties to models that take aggregate averages or expected values. For example, the adoption of new technologies may initially happen in a niche

market. The expansion of such a niche is known to be one way in which the diffusion process starts, but cannot be represented in a model with aggregate markets and a representative firm.

4.14 The enabling environment for innovation – technology infrastructures

New innovations do not typically emerge in a vacuum – the enabling environment which nurtures innovation is also important, including the regulatory environment which determines incentive structures. As Tassey (2008) emphasises, technologies should not be treated as homogeneous entities. The different elements which facilitate innovation have different incentive profiles for investment and should be treated distinctly. Some technology infrastructure has public good characteristics implying spillovers and a tendency for under-investment in the absence of deliberate policy, since the benefits cannot be wholly captured by private investors.

4.15 Demand creation and changes in demand

Demand creation is often an important factor in the commercial success of new technologies, and not infrequently, promising technologies have failed because demand was not generated fast enough to prevent cash flow problems for firms. In the case of technologies with public goods benefits, such as motor vehicle safety or emissions standards, technologies have sometimes failed because governments failed to create demand through regulations or other incentives (Fri, 2003, pp. 68-69). Changes in the structure of demand over time are also important, as the IMCP team concluded (Köhler, 2006, p. 51-52):

> The endogenous growth models do not consider changes in the structure of demand, yet, reduction in energy demand through efficiency measures is a common feature of the energy literature and is represented in several of the IMCP models. … the dynamics of a transition to a low carbon economy is central to climate policy analysis i.e. it is the transition pathways and policies to induce these pathways rather than the very long term equilibrium that matters. Demand led models such as the E3MG model are designed around such analyses, but all models have room to incorporate demand-side responses to efficiency measures (or productivity improvements) as a consequence of ETC and ITC relating to energy end uses and associated dynamics such as rebound effects.

4.16 Induced institutional innovation

ITC models presume that institutions remain constant – but induced innovation in institutions is a broader aspect of innovation that should also be considered, particularly over the long time frames envisaged in emissions mitigation models (Ruttan, 2002).

4.17 Networks, supply chains and innovation

Modern production systems commonly draw on components sourced from all over the world, and prosperous developing regions often owe their success to a profitable niche in a global supply chain. The location of these successful clusters is due to a variety of factors that typically includes good governance, a healthy and educated workforce, affordable and reliable energy, a sound banking sector, good transport and trade links and openness to new ideas. Major industrial clusters in developing countries often rely on exporting to OECD countries. Modelling such innovative regions therefore requires spatial models able to account for the flows of goods, finance, commodities, raw materials, energy and ideas through spatial networks.

4.18 Technological resilience and the importance of diversity

An appropriate degree of diversity is important for the resilience of natural ecosystems and a similar principle applies to technological ecosystems. Societies that are overly dependent on a very small number of core technologies tend to be more vulnerable to disruption than those with a more diverse technology base. A pertinent example today is the critical dependence of modern economies on oil as the primary liquid fuel. While oil is abundant and cheap, that may appear to be the optimal technology choice (leaving aside climate considerations). But there are growing concerns that once the rate of production peaks, as it must, there could be a rate of production decline and subsequent price increases far exceeding the capacities of oil-dependent economies and agricultural production systems to cope.

Capturing the benefits of technological diversity both for resilience against shocks, and also for the spurs to innovation engendered by competition between technologies, requires dynamic models with heterogeneous technologies and even more heterogeneous firms using and developing technologies.

4.19 Uncertainty and innovation

As we have been reminded through the recent financial crisis, uncertainty can have dramatic economic consequences, particularly when it becomes systemic. In seeking to model innovative regions, important sources of uncertainty include the policy credibility of governments, and even the degree of political polarisation in a country. Identical policies can have dramatically different effects depending on the broader context. In one country the policy may be enacted by a strong, credible government with a history of sticking to its decisions, enforcing regulations and having either a strong likelihood of remaining in power or facing an opposition with views not dramatically dissimilar to its own. Either way, investors know the new policy will be enforced. In another country the exact same policy could be promulgated by a weak government with a history of policy back-flips, a track record of non-enforcement of existing laws, with unstable support and facing an opposition with a radically different agenda. Investors have little reason to change their behaviour on this basis of this latest policy whim.

Where it has been introduced, uncertainty has had significant impacts on model behaviour, as Baker and Shittu (2008, p. 2817) attested: “Until recently endogenous technical change and uncertainty have been modelled separately in climate policy models. … Taken as a whole the literature indicates that explicitly including uncertainty has important quantitative and qualitative impacts on optimal climate change technology policy.” This is significant because the current generation of models, which are playing important roles in shaping current policy debates do not adequately incorporate uncertainty. As Köhler *et al.* (2006, p. 48) concluded, “Incorporating uncertainty will be a major challenge for the current generation of climate-economy models.”

4.20 Modelling and policy design for an ‘innovation emergency’

A potentially important and apparently relatively unexplored field, is how to model policies for rapid innovation under emergency conditions. With increasing concern about the consequences of climate change, and the uncertain prospects for a comprehensive global agreement to bring emissions down to a level, and in sufficient time, to avoid dangerous climate change, the possibility grows that drastic emergency measures may be needed in the future. Gradual reforms, relying on market mechanisms to incorporate the real costs of emissions into prices, may not deliver enough innovation fast enough. The only historical analogues we have for the rapid, deliberate economic transformations that may be required, are the general mobilizations undertaken by some countries during war time. Fri (2003, p. 51) observed that:

> In some circumstances, public policy intervention to overcome obstacles to innovation may be justified to secure public benefits. One obstacle is that innovators may be unable to capture all of the available economic benefits of innovation. Another is that economic benefits may not be available and the value of the public good has not been internalized in the market. Experience with energy innovation suggests government intervention works best when it is carefully targeted on specific obstacles.

Under emergency conditions, governments would need to play a more active role, so models capable of illuminating the consequences of different policies in such conditions would require the capacity to model a large number of heterogeneous firm-government interactions, policy instruments, regulations, quotas, incentives and so on.

4.21 Conclusions on modelling innovation and technical change

From the preceding highly condensed discussion, it can be seen that there remains much work to be done. While some models are of course better than others, not one attempts to take account simultaneously of all the issues raised in this section – let alone those raised in Section 3. The IMCP team succinctly summarized their assessment of the state of the art at the time (Köhler, 2006, p. 51):

> The main limitations of current models are: the lack of uncertainty analysis; the limited representation of the diffusion of technology: the homogeneous nature of agents in the models, including the tack of representation of institutional structures in the innovation process. … There is a pressing need to disaggregate learning curves into engineering elements, tackle the problems of causality and the explanations for the learning curve phenomenon. Technology diffusion, within and across sectors, together with the role of FDI and trade is still poorly represented in the climate economy literature. As emphasized by work in the Schumpeterian tradition of disruptive new technologies, whether and how to incorporate uncertainty, as well as addressing heterogeneous agents are issues requiring further conceptual and empirical work.

A new approach to modelling has emerged in recent years however, which potentially offers a framework to take into account many of the issues discussed so far, namely agent-based modelling.

5. Agent-based modelling of regional innovation

Agent-based models (ABMs) are computer simulations based on object-oriented programming, in which discrete 'agents' (objects) interact in real time with each other and their environment according to certain rules.[6] Agents can represent individuals, households, firms, governments or even land types, pathogens, livestock, power grids etc. ABMs still use mathematics, but the mathematics is embedded in the rules governing agents' properties, behaviours and interactions, instead of governing and restricting the entire system and requiring it to converge to an equilibrium. ABMs permit the economic, social, legal, political, geographic, environmental, epidemiological and ethical dimensions of development policies to be integrated to a far greater degree than is possible with purely mathematical models. Agent-based modelling using object-oriented code libraries is also ideally suited to the development of theory based on taxonomical classification of different system components and their interactions. In short, ABMs potentially offer a far more flexible and powerful framework for modeling innovative regions and climate change policies (Tesfatsion & Judd, 2006; Gimblett, 2002).

[6] See Leigh Tesfatsion's comprehensive website at: http://www.econ.iastate.edu/tesfatsi/ace.htm

ABMs have only been developed in the last couple of decades or so though, and while they are growing rapidly in popularity, ABMs have yet to be implemented at the scale required to shed much light on the interlocking problems confronting humanity. But in my view ABMs offer the most promising framework for integrating the multiple interacting dimensions required for sound policy modelling in the future. I am not alone in this view. Boulanger and Bréchet (2005, p. 349) evaluated six different approaches to modelling sustainable development policy, namely macro-econometric, general equilibrium, optimization, Bayesian networks, system dynamics and multi-agent (agent-based) models. Their conclusion was unequivocal:

> Unambiguously, the most promising modelling approach seems to be the multi-agent simulation model. … It is our opinion that public scientific and R and D policy-makers and advisers should foster their development and use in universities, schools and research institutions.

ABMs are also ideally suited to acting as a bridge between disciplines, an essential feature of integrated modelling for innovation and climate change policy. They have opened up a new interdisciplinary research frontier spanning: anthropology, climate change, combat, development and natural resource management, ecology, economics, epidemiology, finance, geography, innovation and organisation theory, migration, operations research, peacekeeping, political science, terrorism and transport.[7]

There have been numerous applications to the field of innovation (e.g. Dawid, 2006; Albino *et al.* 2006; Gilbert *et al.* 2001, Ma & Nakamori, 2005), and active research is also being undertaken on methodological issues such as ABM design and the verification and validation of ABM results. We may also see a merging of simulation modelling and game playing technologies as large multi-player games are used for social research (Bainbridge, 2007). Younger social scientists and economists are very much at home with object-oriented simulations, having been brought up playing computer games.

A more extensive discussion of the role that ABMs could play in exploring the mathematics of social entities is not possible here. I will however address one persistent, and in my view, misplaced, criticism of ABMs – and that is, that they must estimate too many parameters to be useful.[8] This criticism is misplaced because the parameter estimation problem still exists for more traditional analytically tractable models – but it is often dealt with by *arbitrarily* assigning values of 0 (non-existent) or 1 (perfect) to the parameters, with standard deviation always assumed to be zero.

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[7] For a listing of papers under these subject categories, see: http://www-personal.buseco.monash.edu.au/~BParris/BPAgentBasedModelling.html

[8] This discussion and Table 1 are taken from a more extensive paper on this theme: Parris, B.W., (2008) "Top-Down versus Bottom-Up Models for Economic Policy: Where We Start Determines What We Conclude", Working Paper, Monash University, Melbourne, March, 47 pp.

The assumptions of analytically tractable models can all be recast as assignments of parameter values, as in Table 1.

Table 1: ***Common assumptions of tractable analytic models recast as assignments of parameter values***

Parameter	*Value*	*Parameter*	*Value*
• Agent's rationality	1	• Proportion of contracts enforced	1
• Agent's information processing capacity	1	• Cost of contract enforcement	0
• Prevalence of mental illness	0	• Ratio of consumption to wellbeing	1
• Prevalence of addictive behaviour	0	• Cost of evaluating choices	0
• Spatial heterogeneity	0	• Firms' barriers to entry	0
• Spatial separation of markets	0	• Proportion of capital employed	1
• Cost of travel between markets	0	• Mobility of capital between countries	0
• Proportion of agents able to access information	1	• Accuracy of expectations	1
• Information search costs	0	• Cost of redeploying labour	0
• Learning costs	0	• Rate of skill loss of unemployed labour	0
• Time cost of processing information	0	• Degree of corruption	0
• Heterogeneity of preferences	0	• Proportion of firms on the technological frontier	1
• Rate of change of preferences	0	• Time required for consumption	0

The point is that the assumptions of analytically tractable models are not just *like* parameter values, they *are* assignments of parameter values. These arbitrary values are no more scientifically valid than the estimations required for ABMs. Quite the contrary, since we often know that the true parameter values cannot possibly be those that have to be assumed to keep the models tractable.

Ken Judd (2006, p. 887) artfully summarized the inescapable dilemma for economic modellers of all persuasions:

> "[N]umerical errors can be reduced through computation but correcting the specification errors of analytically tractable models is much more difficult. The issue is not whether we have errors, but where we put those errors. The key fact is that economists face a trade-off between the numerical errors in computational work and the specification errors of analytically tractable models."

6. Hybrid modelling using ABMs

As discussed in Section 4.2, there is already a growing tradition of hybrid modelling, and this could be enhanced further by linking ABMs with other species of models. What kind of model is most suitable depends of course on a host of factors including the scale of analysis and the purpose of the exercise.

One alternative is to incorporate an ABM within a dynamic, equation-based model (EBM). In other words, the dynamic EBM 'envelops' the ABM, so that the macrodynamics of the system are governed by the EBM's equations, which impose structure and boundaries on the realisations available to the ABM. This approach could be useful for a system that was suitable overall for modelling as a dynamic EBM framework, but where certain subsystems were more appropriately modelled as ABMs – such as the dynamics of particular firms within overall industry boundaries set by the EBM. For example, the input-output data for a particular industry could be disaggregated across a distribution of firms, so that the total inputs and outputs remain true for the industry as a whole, but individual firms have heterogeneous inputs and outputs, reflecting differences in both firms' sizes and their technologies.

One of the biggest challenges of incorporating an ABM sub-system into an EBM, rather than simply passing information one-way, would be governing the information exchange between the two systems. Most EBMs are designed to produce a unique solution to a system of equations. This implies that any information being passed back to the EBM must be such that it can still converge to a unique solution. In other words, the ABM would have to be restricted in the possibilities which could emerge from it. This could be achieved through the choice of structural equations or behavioural rules, through a particular choice of parameters, or through particular adding up constraints. In any case, the requirement to pass only information that permits convergence to a unique equilibrium solution by the EBM suggests that while the ABM might be a useful add-on, it would not be being used to its full potential.

An alternative is to have the ABM envelop the EBM. The model would be an ABM in its overall architecture, but certain subsystems would be modelled as EBMs in order to reduce model size and computational burden. For example, in an open economy model where our interest lies in the dynamics of agricultural innovation and poverty among peasant farmers in response to agricultural liberalisation, we may want the innovation and income distribution responses to be modelled as an ABM, along with the overall national economy, but the rest of the world, and the changes in global commodity prices might well be more appropriately approximated with an EBM.

7. Quantitative versus qualitative accuracy?

We have seen that there are pervasive uncertainties in model specification, parameter estimation, and data quality and availability – and that model results are highly sensitive to these uncertainties. The question arises: Are there diminishing returns to trying to create more and more 'realistic' data-driven models, if a fixation on data accuracy prevents us incorporating features of the system that are known to be important, but for which data is poor?

The role of significant figures is an important consideration here. The number 3.14159, with six significant figures, is obviously considerably more precise than 3.1 with only two. In scientific research it is generally considered poor form to report results with more significant figures than are available in the lowest quality data that were critical for the results. What are the implications of this principle for economic and social modelling– and indeed for the mathematics of social entities more generally? When certain critical data are of poor quality, how many significant figures should be used to report model results? How can we avoid spurious precision, yet still say something meaningful? This is a different issue to sensitivity analysis. It concerns the legitimacy of reporting more significant figures in the results than are warranted by the quality of the data.

If we find that we can only legitimately present results to one or two significant figures because of irreducible uncertainty in certain critical data, how should that affect our modelling strategy? Perhaps it implies that it might be more important to incorporate the right system components interacting dynamically within plausible parameter ranges, than to aim for a spurious precision by incorporating only those features for which high quality data is available and thereby falling into the trap of model specification errors. But to incorporate the right system components requires a demarcation of the system boundaries and sound aggregation choices. These system boundaries and aggregation choices cannot be deduced axiomatically. They require expert interdisciplinary domain knowledge and sound judgement, and there will often not be a single right answer.

The literature on Earth system Models of Intermediate Complexity (EMICs) seems to parallel some of the issues discussed here. Claussen *et al.* (2002) for example, proposed a new indicator for comparing models which they termed 'integration', which "characterizes the number of interacting components of the climate system being explicitly described in a model":

> In some EMICs, the number of processes and/or the detail of description is reduced for the sake of simulating the feedbacks between as many components of the climate system as feasible. Others, with a lesser degree of interaction, or "integration", are used for long-term ensemble simulations to study specific aspects of climate variability.

8. Conclusion: The place of modelling in policy development

While the mathematics of social entities is undoubtedly interesting from a purely academic perspective, it is also an area requiring urgent attention to inform policy responses to the interlocking challenges of climate change, energy security, food prices, water availability and potential conflict. We need better, more integrated models to inform scenario planning and to help design policy instruments and even new institutions to help us deal with these challenges.

Mathematics is essential, but as a servant not a master. The closed-form, simple analytic models beloved of pure economic theory are of diminishing value in shedding light on the complex interlocking challenges we face. By contrast, mathematics can play a critical role in guiding the design and implementation of sophisticated simulation models, analysing raw input data, sampling parameter spaces and analysing output data.

In the face of a number of serious interlocking challenges, how should we design 'good enough' policies in face of deep complexity, skills shortages, data paucity, irreducible uncertainty and time constraints? In my view Nick Stern was right to emphasise the importance of ethics and risk assessment in policymaking. In his Richard T. Ely lecture to the American Economic Association in January 2008, Stern warned that modelling was useful for illuminating certain aspects of the problem, but that it should not be the primary focus: "This type of modeling does have an important supplementary place in an analysis, but all too often it has been applied naively and transformed into the central plank of an argument" (Stern, 2008, p. 3).

To put some flesh on Stern's argument, consider the following scathing comment and thought experiment from Vaclav Smil (2006, pp. 3 & 5):

> Energy forecasts are not worth even the cost of the cheapest acid paper on which they get printed: even that poor paper will get embrittled only after decades, while most energy forecasts are obsolete in a matter of years, sometimes in just a few months. This conclusion applies equally to technical predictions, price projection or demand aggregates. … I stopped collecting these delusions long time ago, but two recent ones are pretty irresistible. The IPCC (2006) sequestration report assures us that in 2095 it will cost US$ 130 to get rid of 1 t of CO_2: you can best appraise the chances of this being anywhere near the real cost by imagining that it is the year 1917 and you are forecasting a cost of a large-scale commercial technique in the year 2006 on the basis of a purely conceptual outline. And OPEC published its crude oil price forecast for the next two decades – a steady decline to US$ 20/bbl by the year 2025 – just a few months before the prices took off on their climb past US$ 70/bbl.

The mathematics of social entities, and the models arising from the study of this field, have an important role to play in furthering our understanding and in informing policy. But we should be cautious in assessing the possibly spurious mathematical precision of model-based arguments which purport to 'optimize' certain policy responses, particularly in response to complex challenges with enormous risks such as climate change. Models are most useful when considered in the context of robust risk assessment, explicit ethical considerations and expert interdisciplinary domain knowledge.

Acronyms

ABM	Agent-Based Model
AEEI	Autonomous Energy Efficiency Index
BU	Bottom-Up
CGE	Computable General Equilibrium (model)
DSGE	Dynamic Stochastic General Equilibrium models
EBM	Equation-Based Model
EMICs	Earth system Models of Intermediate Complexity
EMF	Stanford Energy Modeling Forum
ES	Energy System (model)
ETC	Endogenous Technical Change
IAM	Integrated Assessment Model
IMCP	Innovation Modeling Comparison Project
ITC	Induced Technical Change
ME	Macroeconometric (model)
OG	Optimal Growth (model)
TD	Top-Down

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